



Consiglio Nazionale delle Ricerche

A program for performance assessment  
of corrugated waveguide transmission lines

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FP18/01

March 2018

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## 1. INTRODUCTION

Long corrugated waveguide transmission lines are commonly used in nuclear fusion experiments, both for heating and diagnosing plasmas.

A common task for designers is computing the polarization along the waveguide and the power transmission, including ohmic and mode conversion loss.

Moreover, designers need often specify alignment and positioning tolerances.

The program described in this report aims at fulfilling these needs and be of use to microwave engineers facing these tasks.

Since polarization calculation is independent on the rest, the program can also be used for computing on-axis polarization along quasi-optical transmission lines.

It is freeware and available in Fortran 90 source form at IFP's website alongside this report.

Users are kindly requested to cite this report in their papers.

## 2. CONCEPTS

### 2.1 TRANSMISSION LINE REPRESENTATION

A transmission line is represented as an ordered sequence of 3D *points* in a single Cartesian reference frame. Each point identifies an *element*. The first and last points in the sequence are the input and output apertures of the transmission line, the other elements are the centres of the mirrors of mitre bends. A *branch* joins an element with the subsequent one. A branch can be made of one or more *pieces* of corrugated waveguide.

A piece of corrugated waveguide has a minimum and a maximum length, the former dictated by assembly considerations, the latter by manufacturing procedures. The number of pieces in a branch is automatically computed, using as many pieces as possible with maximum length, plus a shorter one. If this last piece is too short, then one fewer is used at maximum length, and two identical pieces are inserted to cover the residual distance.

### 2.2 POLARIZATION

Reflection at a mirror is easily represented in an intrinsic reference frame, shown in Figure 1.

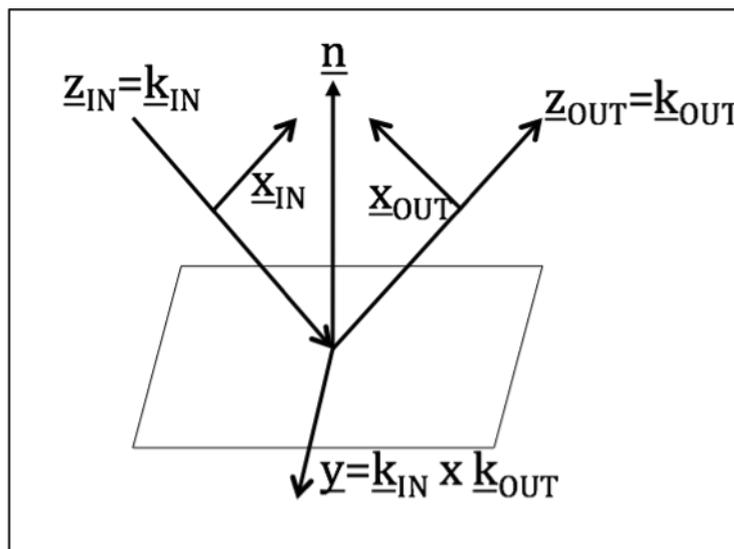


Figure 1: intrinsic reference frame for reflection at a mirror

Using this reference frame, the boundary conditions for specular reflection can be written very simply, so that the incident and reflected fields can be written as:

$$\underline{E}_{IN} = E_x \underline{x}_{IN} + E_y \underline{y}$$

$$\underline{E}_{OUT} = E_x \underline{x}_{OUT} - E_y \underline{y}$$

The input polarization at element 1 (input aperture) must be specified by the user in the intrinsic reference frame for reflection at the first mirror, and is computed for the rest of the transmission line. No rotation of polarization is assumed along the corrugated waveguide.

### 2.3 MITER BENDS: MODE CONVERSION

Mode conversion at mitre bends is computed using the well known formula in [1],[2]:

$$\frac{P}{P_o} = \frac{\chi_{HE11}^2}{3\pi^2} \left( \frac{\lambda}{a \sin \theta} \right)^{\frac{3}{2}}$$

where  $a$  is the corrugated waveguide radius,  $\theta$  is the mitre bend angle, i.e. the angle between  $\underline{k}_{IN}$  and  $\underline{k}_{OUT}$ ,  $\chi$  is the root of the characteristic equation for the HE11 hybrid mode

$$F(\chi_{HE11}, ka, Z) = \frac{1}{d_{HE11}} - d_{HE11} \left[ 1 - \left( \frac{\chi_{HE11}}{ka} \right)^2 \right] + \frac{\chi_{HE11}^2}{kaZ} = 0$$

with

$$d_{HE11} = \frac{J_1(\chi_{HE11})}{\chi_{HE11} J_1'(\chi_{HE11})}$$

and

$$Z = \frac{w}{P} \frac{\tan kd}{1 + 2 \frac{\tan kd}{ka}}$$

The corrugation parameters are the depth  $d$ , the period  $P$ , the width  $w$  of the groove (assumed rectangular).

### 2.4 MITER BENDS: OHMIC LOSS

The ohmic loss in a mitre bend mirror depends on the polarization [2]: the fractional power loss due to ohmic dissipation is, for H and E planes,

$$\frac{P_{\Omega,H}}{P_0} = 4 \frac{R_S}{Z_0} \cos \alpha \quad (\text{H-plane})$$

$$\frac{P_{\Omega,E}}{P_0} = 4 \frac{R_S}{Z_0} \frac{1}{\cos \alpha} \quad (\text{E-plane})$$

where  $Z_0$  is the free-space impedance,  $\alpha$  the angle of incidence (i.e.  $\theta/2$ ) and  $R_S$  is the skin resistance of the mirror material,

$$R_S = \sqrt{\pi \nu \mu_0 \rho}.$$

with  $\nu$  the frequency,  $\mu_0$  the permeability of free space,  $\rho$  the material resistivity.

For a general polarization

$$\underline{E}_{IN} = E_x \underline{x}_{IN} + E_y \underline{y}$$

the ohmic loss is computed as

$$\frac{P_{\Omega}}{P_0} = 4 \frac{R_S}{Z_0} \frac{1}{\sqrt{E_x^2 + E_y^2}} \left( E_x \frac{1}{\cos \alpha} + E_y \cos \alpha \right)$$

where the reference frame is the one shown above. This is a first order approximation, but is acceptable for low loss, especially taking into account the large uncertainty in the material resistivity.

### 2.5 CORRUGATED WAVEGUIDE: OHMIC LOSS

Ohmic attenuation for HE11 in corrugated waveguides is computed using the formula given in [3]

$$\alpha_{HE11} = \frac{R_s \chi_{HE11}^2}{2Z_0 (ka)^2 a} \left[ \frac{1 - \left(1 - \frac{w}{P}\right) \sin^2(kd) + \frac{d}{P} + \frac{1}{2kP} \sin(2kd)}{\left(\frac{w}{P}\right)^2 \sin^2(kd)} + 1 \right]$$

where all the symbols have been defined before.

## 3. POSITIONING TOLERANCES

Providing estimates for acceptable positioning tolerances is frequently done by computing the effect of a single defective item and linearly extrapolating to the whole transmission line. While acceptable in principle for small perturbations, this procedure can be easily improved by computing the effect of a set of random perturbations, and iterating the procedure to provide statistically significant information on error distribution.

Since no second order effects are considered (i.e. error-induced coupling between spurious modes and HE11), one could in principle obtain the same result analytically by assuming a normal distribution of perturbations and using standard error propagation.

### 3.1 ERROR DISTRIBUTION AND STATISTICS

The distribution of pseudo-random numbers is uniform (i.e. flat), and can be used as such. A more common normal distribution can be obtained from random numbers using the Box-Muller transform [4]: if  $u_1, u_2$  are a pair of random numbers from the same uniform probability density in the open interval (0,1), then

$$x_1 = \sqrt{-2 \ln u_1} \cos(2\pi u_2)$$

$$x_2 = \sqrt{-2 \ln u_1} \sin(2\pi u_2)$$

is a pair of random variables from a normal probability density with unit variance and zero mean.

Multiplying them by the standard deviation provides the wanted normal distribution.

Performance of the perturbed transmission line is computed many times, and a histogram is extracted. As error estimators, two numbers are computed from the histogram: the fractional transmission values (in dB) which are exceeded in 95% and 67% of cases. These can be interpreted as the worst-case performance with 95% and 67% probability.

Of course the values, that normally represent  $2\sigma$  and  $1\sigma$  intervals for normal distributions, have no such interpretation here, since the resulting distribution is not normal, rather resembling a Maxwellian.

### 3.2 MODE CONVERSION AT TILT

The formulae in [5] were used for power transmission in abrupt tilts at joints

$$\frac{P_t}{P_0} \cong 1 - 4.2 \left( \frac{a}{\lambda} \Delta\theta \right)^2 + 6.3 \left( \frac{a}{\lambda} \Delta\theta \right)^4 + \dots$$

where  $\lambda$  is the wavelength and  $\Delta\theta$  is the tilt angle.

### 3.3 MODE CONVERSION AT OFFSET

A very similar formula from the same reference [5] can be used for power transmission in waveguide offsets at joints

$$\frac{P_t}{P_0} \cong 1 - 2.7 \left( \frac{\Delta r}{a} \right)^2 + 3.2 \left( \frac{\Delta r}{a} \right)^4 + \dots$$

where  $\Delta r$  is the distance between axes of input and output waveguide at the joint.

### 3.4 MODE CONVERSION: CURVATURE COUPLING

Using the formulae in [6] for mode coupling induced by constant curvature and integrating over the length of the curve, one can find

$$\frac{P_t}{P_0} \cong 1 - \frac{1}{(\Delta\beta)^2} \left( \frac{2\chi_1\chi_2ka}{(\chi_1^2 - \chi_2^2)^2} \frac{1}{R} \right)^2 \left| 1 - e^{j\Delta\beta L} \right|^2$$

where  $\Delta\beta$  is the difference in propagation constants between HE11 and the spurious modes,  $\chi_{1,2}$  are the roots of the characteristic equation for HE11 and spurious mode, R is the radius of curvature and L is the length of the constant curvature section. While  $\chi_1$  is computed exactly from the corrugated waveguide parameters, the constant value 3.8317059702075123 was used for for the spurious modes TE01, TM02 and balanced HE21. Among those that can be excited by constant curvature coupling, these modes are the ones whose propagation constants are closest to that of HE11.

## 4. ERROR MODEL

The design goal was to keep the program usage simple.

As such, only one parameter is used for errors, and it controls the distribution of random displacements of waveguide joints from the ideal position.

Two alternative models are implemented.

### 4.1 TILT PLUS CURVATURE

In the first error model, waveguide joints are supposed to be perfect (i.e. no tilt or offset), and their random displacements bend the waveguides. Waveguide positions are fitted with parabolic splines in both transverse directions (with respect to the axis of the transmission line). The parameters of the splines are used for computing the local radius of curvature, defined as

$$\frac{1}{R} = \frac{|\dot{x} \times \ddot{x}|}{|\dot{x}|^3}$$

The curvature is assumed constant between midpoints of contiguous waveguide pieces.

The random orientations in the last waveguide pieces of two branches joining at a mitre bend are modelled with a localized tilt.

The transmission line is thus a sequence of branches with random undulations, connected at mitre bends with random tilts.

## 4.2 TILT PLUS OFFSET

In the second error model, waveguide pieces are considered perfectly straight, with random transverse positioning errors at both ends. As a result, joints are affected by a random offset, and the tilt is due to the combination of displacements at both ends of every piece. The transmission line is thus modelled with a discontinuous sequence of straight pieces with random orientation.

## 5. PROGRAM STRUCTURE

This section is only intended for those wishing to make modifications to the program and can be safely skipped by everybody else. The program is heavily commented, so this section is kept to a minimum.

Several PARAMETER constants are used for array dimensioning:

- MAXELEM=100: maximum number of elements (i.e. mitre bends/mirrors+input/output apertures)
- MAXPIECES=50: maximum number of waveguide pieces in a single branch
- NSETS=32767: number of sets of random displacements to consider to build the histogram
- NBINS=101: number of bins in the min-max range for computing the histogram

The transmission line is represented with an array, *points(3,MAXELEM)*, containing the user-input coordinates in an ordered list.

Using these data, all the local coordinate systems are computed, together with distances between mirrors/mitre bends.

The former data, with the user-provided input polarization, are used to compute the polarization along the transmission line and the unperturbed transmission line performance.

Using the minimum and maximum length of waveguide pieces, the composition of all branches is computed from the distance between elements, and stored in array *ccwlen(MAXELEM,MAXPIECES)*. Each cell of this 2-D array contains the length of a waveguide piece. The number of pieces per branch is included in *npieces(MAXELEM)*.

This information is used to set-up the computation of random perturbations.

If the user did not set zero tolerance, an array of random displacements is computed, with two values (x,y) per joint and three (x<sub>1</sub>,y,x<sub>2</sub>) for mitre bends, with the two x<sub>i</sub> values for the two branches. The perturbed performance is computed NSETS times, and results in dB are stored in *errdb(NSETS)*.

This array is then processed to extract the normalized histogram and the probability density function.

## 6. PROGRAM USAGE

### 6.1 INPUT DATA

The program reads a single (text) file of parameters, named *xmitline5.txt*. A sample is shown below.

```
60      ;Frequency[GHz]
3.4E-8 ;resistivity of MitreBend mirrors (1.7E-8=Cu, 3.7E-8=7075F Al)
31.75 1.25 1.5 .5 7.4E-8 ;corr w/g parameters: radius,corr.depth,period[mm], corr
                                width/period, resistivity [Ohm.m]
300. 2000. ;minimum and maximum length of a single w/g piece [mm]
1 1 0. ;err.distrib.(0:unif,1:norm),tilt+(0:offset,1:curvature),error[mm](+/-max
                                for unif, stdev for normal)
1. 1. ;input polarization at point 1: E_vector (x,y) (need not be normalized)
```

-59.49 -236.22 -1266.74 ;list of points (centres of mirrors)  
 (...)  
 -29697.18 -12261.66 1394.00

In a more formal specification

Line number	Parameter(s)
1	Frequency [GHz]
2	Resistivity of mitre bend mirrors [ $\Omega$ .m]
3	Corrugated waveguide parameters: radius [mm], corrugation depth [mm], period [mm], ratio of corrugation width to period, resistivity [ $\Omega$ .m]
4	Minimum and Maximum length of a waveguide piece [mm] [mm]
5	Error model: distribution (0 for flat, 1 for normal) model (0 for tilt+offset, 1 for tilt+curvature) error [mm] (+/- range for flat distribution, standard deviation for normal distribution)
6	Input polarization at first point. In the local X,Y coordinates (see 2.2). Values need not be normalized to 1. 1,0 is X, 1,1 is 45deg, 0,1 is Y etc.
7 ff.	Sequence of points in global coordinate system, one point (3 coordinates) per line, with as many lines as necessary. [mm]

**Table 1: structure of input file and explanation of parameters.**

The user may choose to avoid assessing random errors, which is simply done by setting 0. as the error. This is intercepted in the program and greatly speeds up execution. Neglecting Ohmic attenuation in mitre bends and waveguides can be done in the same way, by setting 0 for the resistivity. This does not give any speed enhancement though.

## 6.1 OUTPUT FILES

Three output files are written, *output.txt*, *coord.txt* and *histogram.txt*. Both files are tab-separated text files with header.

### *output.txt*

This is a tab-separated list (with header line) with the following information on each line:

- point number: label of the element, from 1 to the total number
- input polarization [deg]: polarization before the element under consideration. For point 1 it is the user-input polarization. Frame of reference for polarization is the one shown in Figure 1.
- output polarization [deg]: polarization after the element under consideration. For last point it is the output polarization.
- Distance to next element [mm]: length of transmission line connecting current element to next. Empty for last point.
- Fractional Ohmic Loss at mitre bend [dimensionless]. Empty for first, last points.
- Transmission, accounting for Ohmic Loss, at mitre bend [dB]. Empty for first, last points.
- Fractional coupling loss at mitre bend [dimensionless]. Empty for first, last points.
- Transmission, accounting for coupling loss, at mitre bend [dB]. Empty for first, last points.

- Fractional Ohmic loss in waveguide section to next point [dimensionless]. Empty for last point.
- Transmission, accounting for Ohmic Loss, along waveguide branch to next element [dB]. Empty for last point.
- Three columns, containing the normalized x,y,z components of the output electric field in the global reference frame (i.e. the frame of reference where the coordinates of points along the transmission line were given).

### ***coord.txt***

This file is simply a tab-separated list with header, including point number and the components of input, output local coordinate systems in the global reference frame. The y-axis is common for input and output local coordinate systems, so it is only written once.

### ***histogram.txt***

This file includes header lines with the input tolerance parameter and the two error estimators, i.e. the 95% and 67% confidence intervals. Then a line with column headings, and a list of bin centre [dB], fractional number of cases (i.e. number of cases divided by total number of sets), probability density function (pdf). This last is normalized as

$$1 = \int_0^1 f(X) dX$$

where  $X$  is the fractional transmission of the line and  $f$  is the pdf.

## **7. LIMITATIONS**

The program has many limitations, due to the effort to keep its usage simple:

- the error model is limited and, at present, there is no way to treat all three mode conversion mechanisms together
- since using a single tolerance was a design choice, it is not possible to provide separate tolerances for the three mode conversion mechanisms
- the curvature coupling accounts for short random bends, but only considers nearby modes, that have a longer beating length with HE11. This is somewhat contradictory, and may lead to underestimation of curvature coupling. On the other side, this coupling mechanism is of lesser importance for any decently aligned transmission line.

All these limitations can be overcome fairly easily, at the expense of added complexity in usage.

At present, once an applicable error model is chosen, a few (at least three) trials with different tolerances provide sufficient data for fitting analytic models (e.g. parabolas) to confidence intervals, thus allowing to compute the required tolerance, given a requirement on performance degradation.

## 8. REFERENCES

- [1] J. L. Doane, C. P. Moeller (1994): *HE11 mitre bends and gaps in a circular corrugated waveguide*, International Journal of Electronics, 77:4, 489-509, DOI: [10.1080/00207219408926081](https://doi.org/10.1080/00207219408926081)
- [2] M. K. Thumm, W. Kasparek, *Passive High-Power Microwave Components*, IEEE Transactions on Plasma Science, Vol. 30, No. 3, June 2002, p. 755, DOI: [10.1109/TPS.2002.801653](https://doi.org/10.1109/TPS.2002.801653)
- [3] E. A. Nanni et al, *Low-loss Transmission Lines for High-power Terahertz Radiation*, J. Infrared Millimeter and Terahertz Waves (2012) 33:695–714, DOI: [10.1007/s10762-012-9870-5](https://doi.org/10.1007/s10762-012-9870-5)
- [4] G. E. P. Box, M. E. Muller, *A Note on the Generation of Random Normal Deviates*, The Annals of Mathematical Statistics, Vol. 29, No. 2 (Jun., 1958), pp. 610-611, Online: <http://www.jstor.org/stable/2237361>
- [5] K. Ohkubo, S. Kubo, H. Idei, M.Sato, T. Shimozuma, Y.Takita, *Coupling of tilting gaussian beam with hybrid mode in the corrugated waveguide*, International Journal of Infrared and Millimeter Waves 18, No. 1, 1997, pp. 23-41, DOI: [10.1007/BF02677895](https://doi.org/10.1007/BF02677895)
- [6] J.L. Doane, *Propagation and mode coupling in corrugated and smooth-wall circular waveguides*, in *Infrared and millimeter waves*, Volume 13 (A86-37151 17-33). Orlando, FL, Academic Press, Inc., 1985, p. 123-170 ISBN 0323152775. Online: <https://books.google.com/books?id=LndVIm0elCoC&pg=PA123>