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A small program for writing hybrid mode fields in corrugated waveguides in GRASP-compatible format

A.Simonetto

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ISTITUTO DI FISICA DEL PLASMA
"Piero Caldirola"

Via R. Cozzi 53 - 20125 Milano (Italy)

Introduction

GRASP[1], the well-known program for antenna simulation, includes hybrid modes of circular corrugated waveguide as a field source. But only balanced modes are computed, being those of widest interest for antennas.

Studying radiation of hybrid modes in very broadband applications (e.g. reflectometry in fusion experiments) often requires to cope with unbalanced conditions at band extrema.

A FORTRAN90 program was written for writing the field of a chosen hybrid mode on the aperture of a circular corrugated waveguide in a format compatible with *tabulated sources* in GRASP[3].

The formulation was taken from [2].

The program is available in the public domain as a *.tar.gz* archive at the same location as this report.

Formula summary

The three components of the electric field of the hybrid mode are written in cylindrical coordinates [2] as

$$\underline{E}_{pm}(r,\varphi)=\sqrt{R_{pm}Z_0}\left[\underline{e}_r\left(\partial_r T_{pm}+\frac{d_{pm}}{r}\partial_\varphi T'_{pm}\right)+\underline{e}_\varphi\left(\frac{1}{r}\partial_\varphi T_{pm}-d_{pm}\partial_r T'_{pm}\right)+\underline{e}_z\left(j\frac{X_{pm}^2}{R_{pm}a\,ka}T_{pm}\right)\right] \quad (1)$$

$$\underline{H}_{pm}(r,\varphi)=\frac{1}{\sqrt{R_{pm}Z_0}}\times$$
$$\times\left[-\underline{e}_r\left(\frac{1}{r}\partial_\varphi T_{pm}-d_{pm}R_{pm}^2\partial_r T'_{pm}\right)+\underline{e}_\varphi\left(\partial_r T_{pm}+\frac{d_{pm}R_{pm}^2}{r}\partial_\varphi T'_{pm}\right)+\underline{e}_z\left(j\frac{d_{pm}R_{pm}X_{pm}^2}{a\,ka}T'_{pm}\right)\right] \quad (2)$$

with

$$T_{pm}(r,\varphi)=N_{pm}J_p\left(X_{pm}\frac{r}{a}\right)\sin p\varphi \quad (3)$$

$$T'_{pm}(r,\varphi)=N_{pm}J_p\left(X_{pm}\frac{r}{a}\right)\cos p\varphi \quad (4)$$

The quantity d_{pm} represents the ratio between TE and TM components in the normal mode

$$d_{pm}=\frac{pJ_p(X_m)}{X_m J'_p(X_m)} \quad (5)$$

The quantity R_{pm} is the normalized propagation constant

$$R_{pm}=\frac{\beta_{pm}}{k}=\sqrt{1-\left(\frac{X_{pm}}{ka}\right)^2} \quad (6)$$

X_{pm} is the root of the characteristic equation

$$\frac{1}{d_{pm}}-d_{pm}R_{pm}^2+\frac{X_{pm}^2}{pkaZ_{eff}}=0 \quad (7)$$

Z_{eff} is the longitudinal effective wall impedance

$$Z_{eff} = \frac{w}{P}\frac{\tan kd}{1+\frac{2}{ka}\tan kd} \quad (8)$$

and the normalization constant N_{pm} is

$$N_{pm} = \frac{1}{X_{pm}J_p\left(X_{pm}\right)\sqrt{\frac{\pi}{2}\left[\left(1+R_{pm}^2 d_{pm}^2\right)\left(1-\frac{p^2}{X_{pm}^2}+\frac{2p}{d_{pm}X_{pm}^2}+\frac{p^2}{d_{pm}^2 X_{pm}^2}\right)-2\left(1+R_{pm}^2\right)d_{pm}\frac{p}{X_{pm}^2}\right]}} \quad (9)$$

The *balanced* condition occurs when Z_{eff} is infinite, ie. when $kd=\pi/2$.
Writing the cartesian components of Epm as

$$\begin{aligned} E_{x,pm} &= E_{r,pm}\cos\varphi - E_{\varphi,pm}\sin\varphi \\ E_{y,pm} &= E_{r,pm}\sin\varphi + E_{\varphi,pm}\cos\varphi \end{aligned} \quad (10)$$

one can finally write for the electric field

$$\begin{aligned} E_{pm} = \frac{N_{pm}}{a}\Bigg\{ & -\sqrt{R_{pm}Z_0}\,p\frac{\left(\left(1+d_{pm}\right)J_p\left(X_{pm}\rho\right)-d_{pm}J_{p-1}\left(X_{pm}\rho\right)\frac{X_{pm}\rho}{p}\right)}{\rho}\cos p\varphi\sin\varphi + \\ & -\sqrt{R_{pm}Z_0}\,p\frac{\left(\left(1+d_{pm}\right)J_p\left(X_{pm}\rho\right)-J_{p-1}\left(X_{pm}\rho\right)\frac{X_{pm}\rho}{p}\right)}{\rho}\cos\varphi\sin p\varphi, \\ & \sqrt{R_{pm}Z_0}\,p\frac{\left(\left(1+d_{pm}\right)J_p\left(X_{pm}\rho\right)-d_{pm}J_{p-1}\left(X_{pm}\rho\right)\frac{X_{pm}\rho}{p}\right)}{\rho}\cos p\varphi\cos\varphi + \\ & -\sqrt{R_{pm}Z_0}\,p\frac{\left(\left(1+d_{pm}\right)J_p\left(X_{pm}\rho\right)-J_{p-1}\left(X_{pm}\rho\right)\frac{X_{pm}\rho}{p}\right)}{\rho}\sin\varphi\sin p\varphi, \\ & j\frac{J_p\left(X_{pm}\rho\right)X_{pm}^2 Z_0}{ka\sqrt{R_{pm}Z_0}}\sin p\varphi\Bigg\} \end{aligned} \quad (11)$$

where the derivatives of Bessel functions were expanded using recurrence relations [4].

Program description

Installation

The program is distributed as a *.tar.gz* archive, that should be unpacked. Users should change the line terminator to suit their environment (LF for Unix/Linux, CR+LF for Windows) in all text files, using a decent text editor.
It was tested with GFORTRAN. Compilation instructions are given in the README.txt file for Unix/Linux, but they are almost identical in Windows command-line environment.
The one-line instruction for compilation is

```
gfortran -Wall hemngrasp_v3.f90 xmhemn.f90 hemn.f90 -o hemngrasp
```

Input

The program has no interactive input. The input data file is named `input.txt`, and a typical case is shown below:

```
15.    :min freq[GHz]
75.    :max freq[GHz]
3       :number of frequencies
88.9   :WG diameter[mm]
1.67   :corrugation depth[mm]
.75    :w/p corrugation width over period
1 1 1  :hybrid mode to compute (0:EH/1:HE, m,n) (e.g. 0 1 2 for EH12)
```

The minimum and maximum frequencies can be identical. If so, the number of frequencies (third line) must be 1. The lines are commented and should be self-explanatory.

The program also reads a file named `eigenval.txt` that holds circular waveguide modes eigenvalues. This file should never be changed.

Output

Only the number of samples on each row and column and the frequencies are written on screen. Screen output for the sample above is listed below.

```
         101 points at           1  frequencies
 fghz=   15.0000000000000
```

The output file is named `HEmnFLD.grd` or `EHmnFLD.grd` depending on the type of mode and is written in the proper *.GRD* format for GRASP Tabulated Sources. Users should of course change the line termination to suit their GRASP distribution.

The output field is *y*-polarized, unlike the GRASP default, so one should define a suitable corrdinate system in GRASP.

Program validation

The program is quite simple, but validating it is far from trivial. The format of the output file was directly validated with GRASP. All formulas were derived with Mathematica using symbolic substitutions, cross checking them with [2] whenever possible. FORTRAN statements were copied from Mathematica formulas using a text editor to convert the syntax.

Having used this conservative approach in coding, one can be reasonably confident that successful comparison with the built-in GRASP sources under balanced conditions can be used to infer proper program behaviour elsewhere in parameter space.

The tables below show the far field patterns obtained for HE_{11} in corrugated waveguides with internal diameter of 88.9 and 63.5 mm and EH_{12} in 88.9 mm only.

Data are obtained from the built-in Hybrid Mode Conical Horn in GRASP (with 0 deg flare angle) and from a Tabulated Planar Source using the output of this program for waveguides of the same diameters, corrugation depth 1.67 mm and ratio w/P of corrugation width to period equal to .75.

Patterns were computed at three frequencies: 75 GHz ($d/\lambda \approx 0.42$), 45 GHz ($d/\lambda \approx 0.25$, i.e. the balanced condition), 15GHz ($d/\lambda \approx 0.08$).

One can see that the middle row, corresponding to 45 GHz, i.e. the balanced condition, is identical in all cases (HE_{11} in 88.9 and 63.5 mm w/g, EH_{12} in 88.9 mm w/g) within the default calculation accuracy. This provides confidence on program behaviour.

When frequency is increased (75 GHz, third row of all tables) the two results are still not too different in the main beam, especially for the large 88.9 mm diameter. But the difference becomes very large when frequency is moved far below the balanced condition (15 GHz) especially for EH_{12}, that is close to TE_{12} for these parameters.
All of this is expected and is the reason for writing the program: the balanced mode approximation is less and less accurate as one moves away from the resonant condition, more and more so for smaller diameters and smaller w/P.

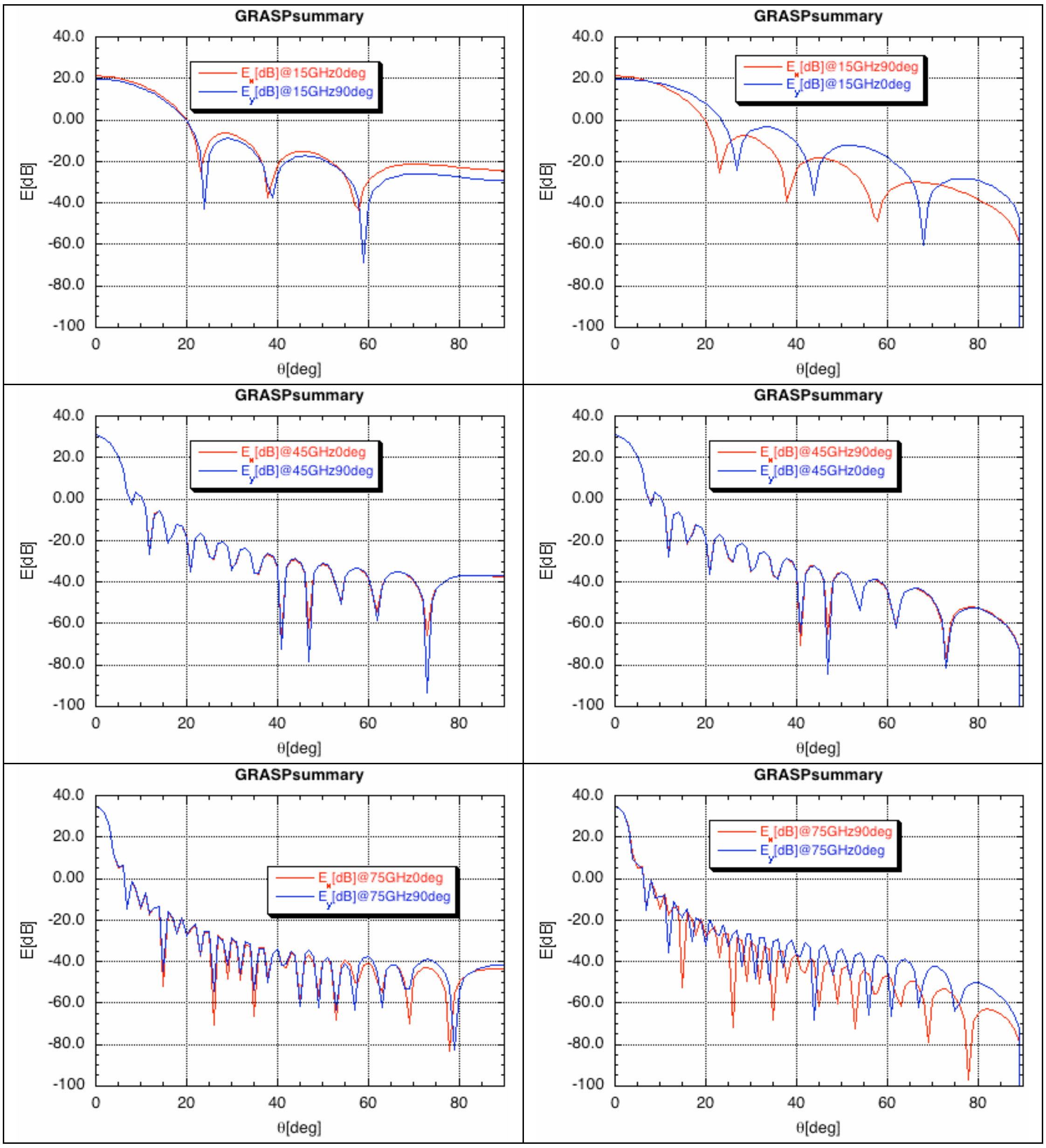

Table 1: HE_{11} far-field patterns (0 to 90 deg in elevation), for 0 deg (left column) and 90 deg (right column) in azimuth. Frequency 15 GHz (top row), 45 GHz (middle row), 75 GHz (bottom row). Red: GRASP Hybrid Mode Conical Horn, blue: Tabulated Planar Source using the output of this program. Corrugated waveguide diameter 88.9 mm, corrugation profile as described in text.

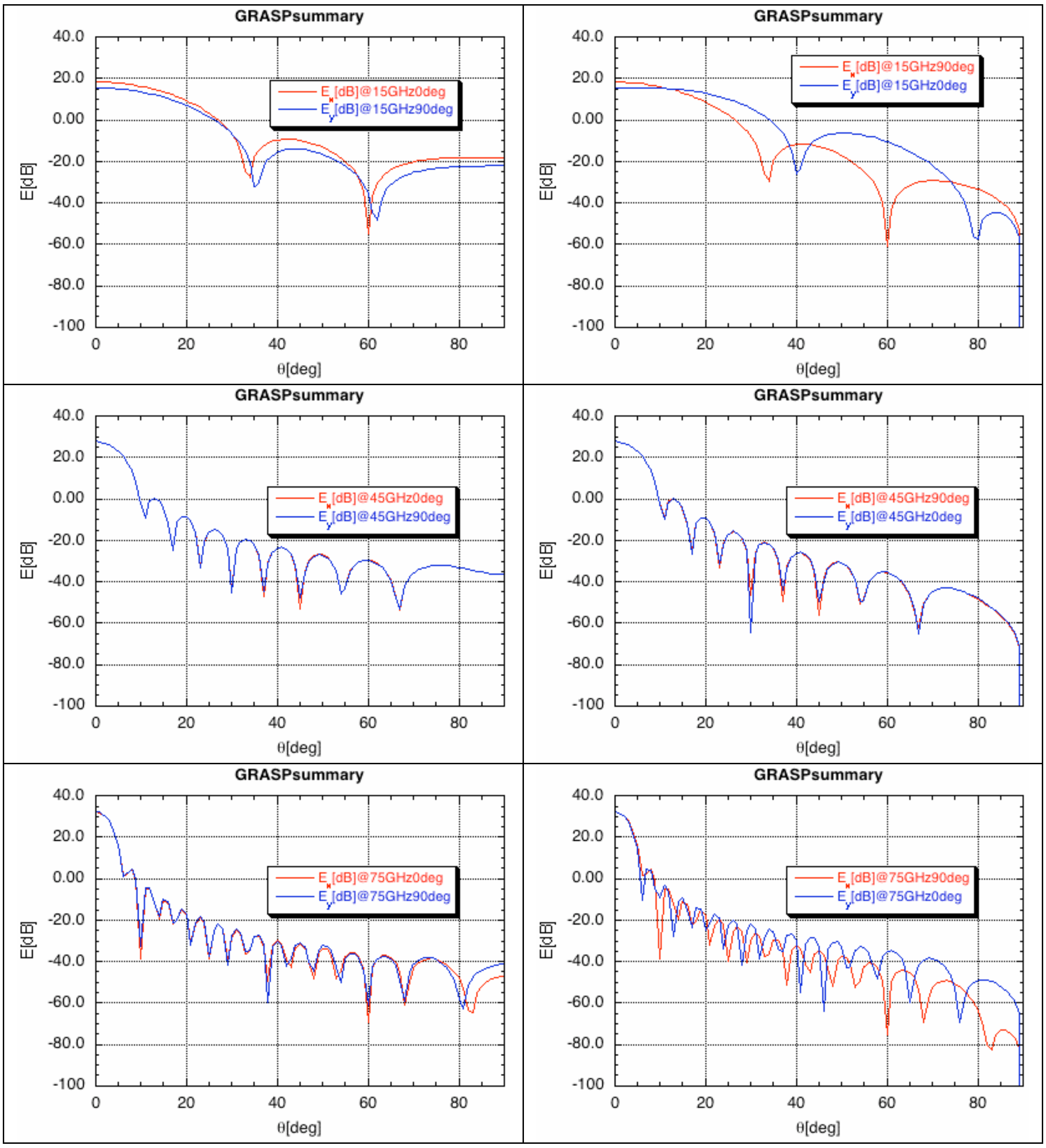

Table 2: HE_{11} far-field patterns (0 to 90 deg in elevation), for 0 deg (left column) and 90 deg (right column) in azimuth. Frequency 15 GHz (top row), 45 GHz (middle row), 75 GHz (bottom row). Red: GRASP Hybrid Mode Conical Horn, blue: Tabulated Planar Source using the output of this program. Corrugated waveguide diameter 63.5 mm, corrugation profile as described in text.

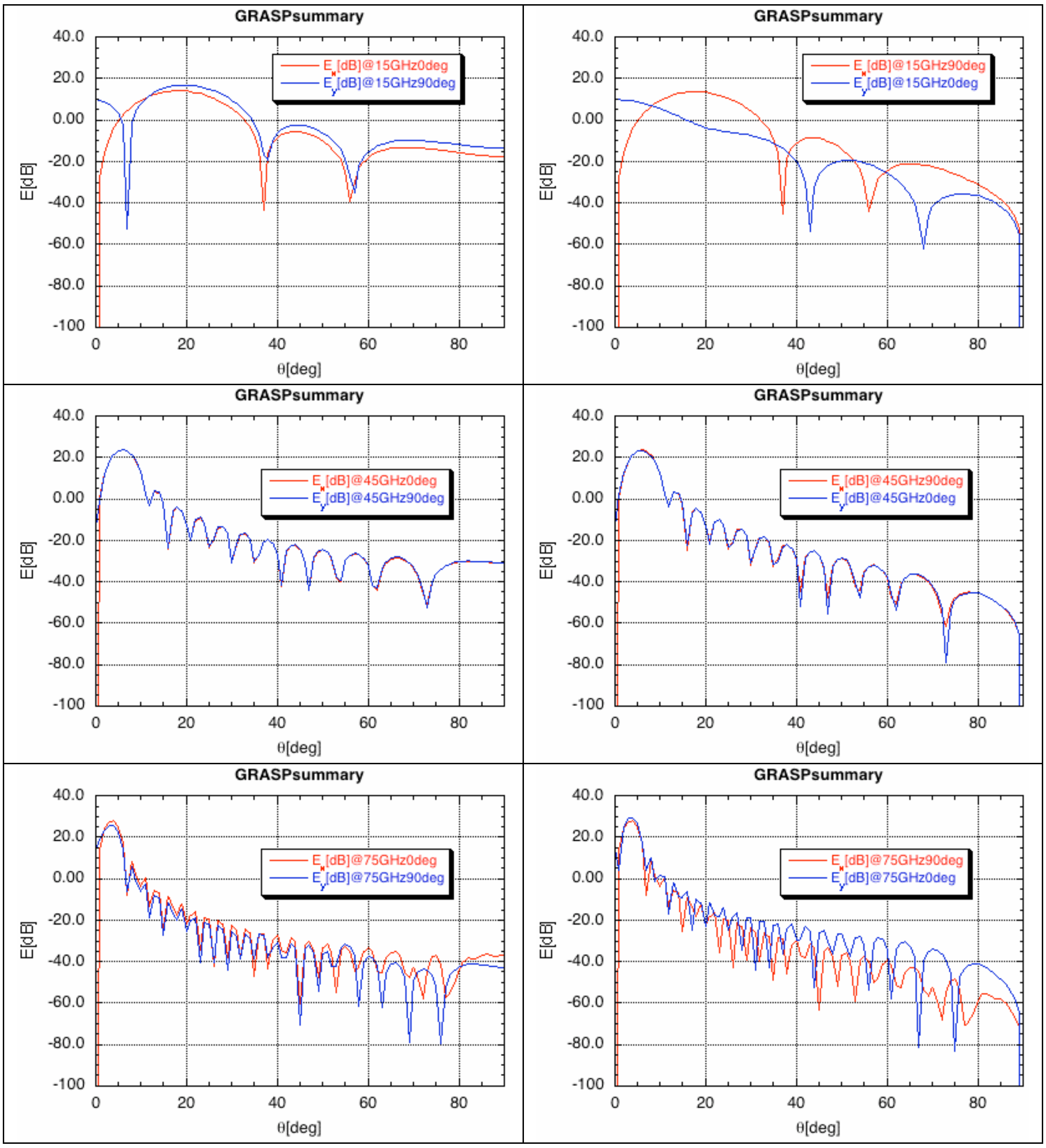

Table 3: EH_{12} far-field patterns (0 to 90 deg in elevation), for 0 deg (left column) and 90 deg (right column) in azimuth. Frequency 15 GHz (top row), 45 GHz (middle row), 75 GHz (bottom row). Red: GRASP Hybrid Mode Conical Horn, blue: Tabulated Planar Source using the output of this program. Corrugated waveguide diameter 88.9 mm, corrugation profile as described in text.

References

[1] http://www.ticra.com

[2] J.L. Doane, *Propagation and mode coupling in corrugated and smooth-wall circular waveguides*, in Infrared and Millimeter Waves, ed. K.J. Button vol. 13, Ch. 5, 1985, pp. 123-170

[3] GRASP 9 Reference Manual, TICRA *"F2.2 Field data in rectangular grid"*

[4] M. Abramowitz,, I. Stegun (eds) *Handbook of Mathematical Functions,* Dover 1967, 9.1.27