Hypervelocity regime of dust particles in tokamaks

Enzo Lazzaro[a], S. Ratynskaia[b],Igor Proverbio[a]

[a] *Istituto di Fisica del Plasma- CNR "P.Caldirola, Assoc. Euratom-ENEA-CNR*
Via R.Cozzi 53, 20125 Milan, Italy

[b]*Royal Institute of Technology, Stockholm, Sweden*

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Abstract. The mobilization and acceleration of metallic dust in the gap region between the last closed confinement surface and the vessel wall of the Frascati Tokamak Upgrade (FTU) is studied numerically for the definition of appropriate location of diagnostics devoted to dust dynamics.

Keywords: ion -dust friction forces, hypervelocity, tokamak, elastic collisions, standard mapping, scrape-off layer
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MOTIVATION AND OBJECTIVES

Dust particulates are commonly found in magnetic fusion devices [1] with a size ranging between ~10 nm and a few hundred micrometres. The toroidal ion edge flow (in the Scrape-Off Layer, or SOL) of a tokamak can provide a tremendous acceleration mechanism up to hypervelocity regimes (several km/s) for dust particles present in the gap between the last closed magnetic surface and the vessel wall.

This can become a serious problem for the wall integrity, the secondary impurity extraction and the tritium inventory. The mobilization and acceleration of metallic (ferromagnetic) dust in gap region between the last closed confinement surface and the vessel wall of a *limiter* tokamak (such as FTU) is investigated numerically. The inspection of the numerical solution of the appropriate Newton equation of motion suggests a similarity analysis to develop scaling arguments and a generic mechanism for a quasi stochastic increase of kinetic energy. This leads to determine scaling law for the maximum achievable speed.

The detailed physical mechanisms governing the motion of dust in the tokamak environment depends on several detailed constraints, determined by the nature of the materials of the first wall and by its specific geometry. This forbids a completely general discussion, however it is possible to isolate basic characteristic of the motion of dust particles in a closed volume, that can be applicable to several other problems.

Basic Phenomenology of Hypervelocity Impacts

According to literature on the impact of metallic particles on metallic walls [2,4], referring specifically to steel (Fe) against steel, a maximum quasi-elastic speed for μm particles should not exceed ~ 1 − 1.5 km/s. For projectile velocities typically larger than a few km/s, stresses arising in bodies on impact considerably exceed the yield point of materials. Under these conditions solids behave like liquids [2]. Impacts at such velocities are defined hypervelocity impacts [3,4] where the resulting pressure can reach 1 TPa and the temperature can be sufficient to melt (formation of solid ejecta), vaporize (formation of cloud of neutrals) the material of the target and the impinging particle and partially ionize the neutrals released (formation of a plasma cloud). This complex phenomenology has been kept in mind in setting appropriate limits in the development of an essential mechanical model of motion of a finite size dust particle, within a tokamak vessel.

Model of Dust Motion between the Plasma Edge and the First Wall of a Tokamak

The typical profiles of magnetic and radial electric field in a tokamak (like FTU) where the Last Closed Magnetic Surface (LCMS) is defined as the (isobar) flux surface in contact with a limiter and is circular, are given in equation 1. The tokamak magnetic configuration is determined by the toroidal field on axis B_0, the aspect ratio and the safety factor q(r):

$$\mathbf{B} = \frac{B_0 R_0}{R}\hat{e}_\phi + \frac{rB_0}{R_0 q(r)}\hat{e}_\theta, \qquad \varepsilon = r/R_0 \tag{1}$$

in the coordinate system (r,θ,ϕ) related to (R,ϕ, Z) by

$$R = R_0 + \Delta_{sh} + r\cos\theta, \quad Z = r\sin\theta, \quad \phi = \phi \tag{2}$$

For the present problem the important plasma profiles are those of the species densities, temperature, radial electric field in the radial region of the Scrape Off Layer (SOL) and the tenuous plasma region up to the metallic vessel wall. In the SOL the plasma parameters and typical exponentially decaying profiles are modeled after ref [5] and used here (see Table I and Fig.1) to set up the reference scenario for a study of the motion of metallic, ferromagnetic, particles that are present in the annular space between the last closed magnetic surface (LMCS) and the vessel wall.

The motion of a spherical dust particle with a diameter 2a in the range 2-20 μm, that may never decrease below 0.5 μm, is governed by the basic forces present in tokamak, namely those due to magnetic field, the peripheral electric field, gravity, the friction forces with plasma ions and neutrals and collision with the wall. The typical dust particle can be considered "isolated", in the sense that $\Delta/\lambda_D = \left(3/\left(4\pi n_d\right)\right)^{1/3}/\lambda_D >> 1$ where Δ is the average spacing between dust grains, and λ_D is the Debye length. However the dust parameters such as grain charge and temperature must be determined by appropriate charging and heating models based on the plasma particles and energy fluxes on the grain.

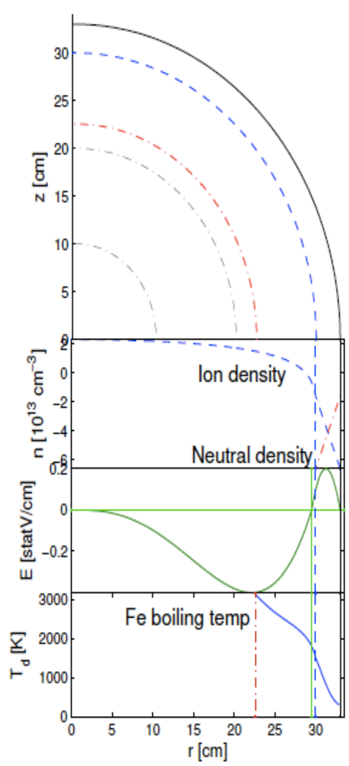

FIGURE 1. (top) Traces of relevant surfaces in the tokamak poloidal quadrant up to and across the LCMS (blue dashed line) (middle) ion and neutral density profiles; (bottom) radial electric field and electron temperature profiles,up to and within the SOL. .

TABLE 1.

a_c	n_e (10^{13}cm^{-3})	n_i (10^{13}cm^{-3})	n_n (10^{13}cm^{-3})	T_e (eV)	T_i (eV)
30	0.2	0.2	0.0005	30	30
0	5	5		500	500

The determination of the motion of a dust grain in the axisymmetric tokamak geometry of aspect ratio R/r >>1 (described in coordinates r,θ,ϕ) is an nonlinear problem requiring the numerical solution of Newton equations which include the magnetic gradient force acting on the ferromagnetic dipole, a heat balance condition on the grain, and boundary conditions of quasi elastic reflections from the wall, up to the hypervelocity regime described above:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v} \tag{3}$$

$$M_d\left\{[\dot{v}_r - \frac{v_\theta^2}{r}]\mathbf{e}_r + [\dot{v}_\theta + \frac{v_r v_\theta}{r}]\mathbf{e}_\theta + [\dot{v}_\phi + \frac{v_\phi v_r \cos\theta}{R} - \frac{v_\phi v_\theta \sin\theta}{R}]\mathbf{e}_\phi - \frac{v_\phi^2}{R}\mathbf{e}_R\right\} = -\mathbf{v}\frac{dM_d}{dt} + \sum_{\alpha-i,n}\mathbf{F}_{fric,\alpha} + \mathbf{F}_E + \mathbf{F}_{\mathbf{v}\times\mathbf{B}} + \mathbf{F}_{\nabla B} + M_d\mathbf{g}$$

The dust particle with its large mass does not feel the Lorentz force but the inertial acceleration slings the particle toward wall as soon as it gains speed. The dominant *interaction* of a dust grain, mobilized from any sticking position on the wall by virtue of the ambient non uniform magnetic field, is the friction force with the plasma ions [1,6] which possess a ordered flow speed corresponding to the EXB drift, at the periphery of the confinement volume which can span a range 5-50 km/s, in a tokamak like FTU [5]:

$$V_\phi \approx \frac{cE_r}{B_\theta} \tag{4}$$

The plasma–dust and neutral–dust friction forces, according to Ref.[6]can be expressed as

$$\mathbf{F}_{fric,\alpha} = \varsigma_\alpha m_\alpha n_\alpha v_{Th_\alpha}\left(\mathbf{V}_\alpha - \mathbf{v}\right)\sigma_d \tag{5}$$

where $\sigma_d = \pi a_d^2$ is the grain geometric area, and for each species $\alpha = i,n$, v_{Th_α} are the thermal speeds, ς_α are friction coefficients reported in [6] as functions of the normalized velocity difference $c_\alpha = \left|\left(\mathbf{V}_\alpha - \mathbf{v}\right)/v_{Th_i}\right|$. All the other interactions are subdominant, except close to the wall where friction with ions vanishes and collision with the wall prevail. The full integration of eqs.3 with data typical of the FTU tokamak and injection at the point marked X, gives trajectories in the equatorial and meridian (poloidal) cross sections shown in Fig.2

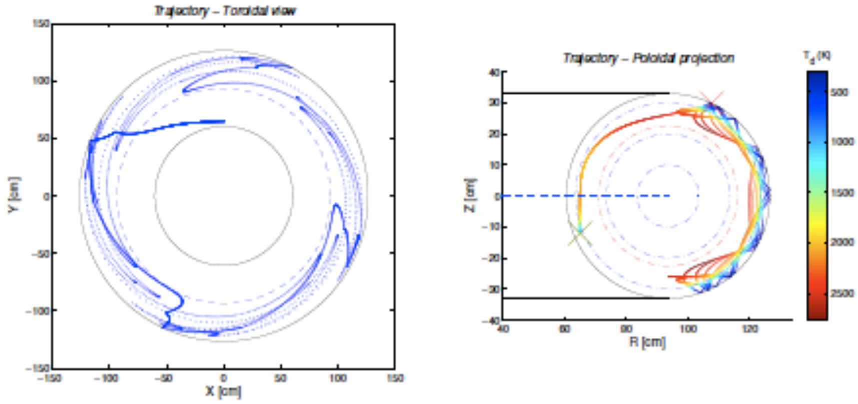

FIGURE 2. (left)projection in the equatorial plane of the FTU tokamak of the trajectory of a 2μ Fe dust paritcle; (right) projection in a meridian (poloidal) cross section

For the example considered, the toroidal speed attained by the dust particle reaches 1km/s with N bounces from the wall, before its temperature reaches the fusion point that terminates its destiny.

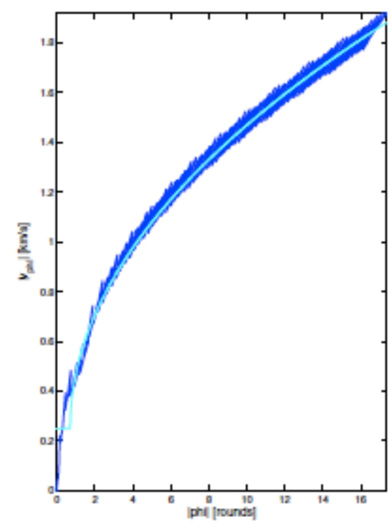

FIGURE 3. For the case of Fig.3 here the dependence is shown of the toroidal dust velocity (dark blue line) on the number N of toroidal loops, compared with the scaling $N^{1/2}$ (light blue line).

It is apparent from these trajectories that the combination of the ion-drag mechanism in a layer within the SOL and the inertial acceleration to the wall where multiple reflections can occur, tend to “trap” the particle within a poloidal angular sector.

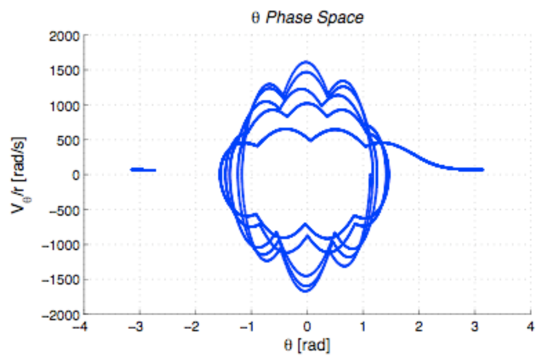

FIGURE 4 Phase space section (V_θ/r, θ) corresponding to the trajectory of Fig.3

An inspection of the motion in the phase space section (V_θ/r, θ) shows significant topological analogy with the phase space of paradigmatic mechanical systems . This observation stimulates a further analysis of the essential dynamics of the system using dimensional arguments and relevant analogies for the construction of a conceptual model, useful for the interpretation of the results.

Conceptual Model of Acceleration Mechanism and Scaling of Maximum Speed Gain

Important aspects of the dynamics described by the full equations (3) can be understood extracting a simplified model with some basic ordering assumptions on the active forces and on the particles and ions velocity components.
We consider in particular :

$$V_{ir} = O(\varepsilon), \quad V_{i\theta} = O(\varepsilon), \quad V_{i\phi} = V_{i\phi 0} \tag{6}$$

and consequently the velocity differences with the ion flow speed turn out to be

$$\Delta v_r \equiv V_{ir} - v_r = O(\varepsilon^2), \quad \Delta v_\theta \equiv V_{i\theta} - v_\theta = -v_\theta + O(\varepsilon), \quad \Delta v_\phi = V_{i\phi} - v_\phi \tag{7}$$

It is also necessary to give expressions of the flight time T between collisions and the projections of interaction length:

$$\delta t \approx \delta R/\delta v_R\,, \quad L_\phi = v_\phi \delta t = \frac{v_\phi}{v_R}\delta r, \quad L_\theta = \frac{v_\theta}{v_r}\delta r \tag{8}$$

It is convenient to introduce dimensionless parameters to express the ion-friction force following the literature [6] and consistent with assumptions (7)

$$M = \frac{v_d}{v_{Th_i}}; \quad m_i v_{Th_i} = \frac{2MT_i}{v_d}; \quad K = \varsigma m_i n_e v_{Th_i} \sigma_d M_d^{-1} = \frac{3\varsigma n_e}{2a_d \rho_d}\frac{MT_i}{|v_d|} \approx \frac{k}{|v_d|}, \quad kr >> 1 \tag{9}$$

The resulting model form of eqs. (3) in the interaction region, where kr>>1,is then:

$$\frac{dv_r}{dt} = \frac{v_\theta^2}{r} + O|\varepsilon^2|; \quad \frac{dv_\theta}{dt} = -\frac{k}{|v|}v_\theta; \quad \frac{dv_\phi}{dt} = \frac{k}{|v|}\left(V_{i\phi} - v_\phi\right); \quad \frac{dv_R}{dt} \approx \frac{v_\phi^2}{R} \tag{10}$$

The interaction is intermittent and periodic due to collisions with the wall, so it is modeled by k=k(t) a periodic pulse function. Here we consider first the case with a constant value $k = const.$.
The conditions (6) and (7) imply that:

$$\frac{dv_\theta}{dt} = -\frac{k}{|v|} v_\theta; \quad \frac{d\Delta v_\phi}{dt} = -\frac{k}{|v|} \Delta v_\phi \tag{11}$$

and therefore

$$v_\theta = \Delta v_\phi + C, \qquad v_\theta = -v_\phi = u, \qquad |v| \cong \sqrt{2}|u| \tag{12}$$

for a suitable choice of the integration constant C. Furthermore this leads to a generic form of equations (10):

$$\frac{d|u|^2}{dt} = -\sqrt{2}ku \tag{13}$$

Equation (13) can be integrated over a flight time T =L/|u| between collisions with the wall. From relations (12) a general scaling is deduced for the velocity that can be attained in one flight :

$$|u| = kt\big|_0^T = \sqrt{2}k\frac{L}{|u|}, \quad |v_\phi| = \left(\sqrt{2}kL\right)^{1/2} \tag{14}$$

From the elimination in (8) of v_R and the estimate of the toroidal interaction length $L_\phi \sim v_\phi \delta t \sim \sqrt{R\delta r}$ one obtains from the first of eqs.(8) the scaling of the maximum toroidal velocity in a *single* flight time:

$$v_\phi \sim \left(k\sqrt{R\delta r}\right)^{1/2} \tag{15}$$

If one allows N quasi-elastic collisions with the wall the estimate (15) should be multiplied by $N^{1/2}$. In a machine with limiter a number $N=O(10^2)$ is possible, while in a machine with divertor configuration it is N=O(1) .This estimate, in excellent agreement with the numerical result shown in Fig.4 can be used for a quick assessment of experimental situations, although it *cannot* replace the full numerical solution of eqs.3.

Phase space analogical modelling

After each collision with the wall the poloidal and toroidal components of velocity of the particle reinjected in the ion-friction layer, depend on the instantaneous angle of reflection from the wall that in turn depends on the poloidal angle θ " trapped" between $-\pi/2$ and $\pi/2$, as shown in Fig.2.

A plausible extension of the "conceptual model", to be validated a posteriori, is to consider that the *intermittent* force k should be described both in terms of periodicity in t and in θ. By renaming $q=r\theta$ $u=v_\theta$ as dq/dt and p as du/dt equation (13) can be cast in the equivalent "formal" presentation:

$$\boxed{\begin{aligned} \frac{dq}{dt} &= p \\ \frac{dp}{dt} &= k(t,q) = -\sum_n^N \delta(t-nT)\cdot\sum_m k_m \sin mq \end{aligned}} \tag{16}$$

Then the analogy with the "kicked rotor" dynamics [9], becomes striking and it is tempting to pursue further this *qualitative analogy* (not a quantitative identification!) with this classical mechanical model to gain a more intuitive understanding of the physical process. The (p,q) phase space sections of Fig.5 associated with the full equations (3) and with the "kicked rotor",respectively, show the *topologically* likeness of the two systems with a layer of nearly random behaviour of (p,q)., for $N\sim10^2$ and k>1.

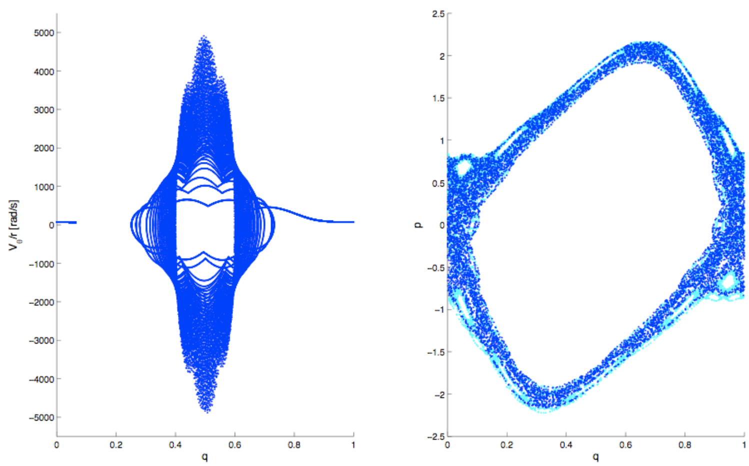

FIGURE 5. Phase space section (V_θ/r, θ) corresponding to the trajectory of Fig.3 extended to1000 collisions, compared with (p,q) phase space of kicked rotor

We recall that in terms of canonically conjugate (dimensionless) variables (p,q) the classical periodically kicked rotor is described by the Hamiltonian

$$H(p,q,t) = \frac{p^2}{2} + k(t)\cos q \tag{17}$$

where k(t) is a train of N pulses. The Hamilton equations integrated over one pulse yield the famous "standard mapping" [7,8] that for |k|>>1 is known to yield a stochastic behaviour so that q then varies wildly, (see Fig.5) and can be treated as a random variable, uncorrelated for different n

$$\begin{aligned} p_{n+1} - p_n &= k \sin q_{n+1} \\ q_{n+1} &= q_{n+1} + p_n \quad mod(2\pi) \end{aligned} \tag{18}$$

It is then possible to evaluate a mean square increase in the momentum p, equivalent to a diffusion coefficient [9]:

$$\left\langle (\Delta p_n)^2 \right\rangle = k^2 \left\langle (\sin \vartheta_{n+1})^2 \right\rangle = \frac{k^2}{2} = 2D \tag{19}$$

Using a classical random walk argument in momentum space, an evaluation can be made of the mean momentum increase, using a diffusive distribution function:

$$f(p,N) = (2\pi ND)^{-1/2} \exp\left[-\left(p^2/2ND\right)\right], \quad \overline{p^2/2} = \int p^2 f dp \sim DN \tag{20}$$

From the correspondence statements (12) and (16) this latter result appears to scale exactly as that predicted from eq, (15), multiplied by $N^{1/2}$ collisions with the wall and with Fig.4.

CONCLUSIONS

A full calculation of the motion of a ferromagnetic dust particle in a tokamak ith limiter has shown the possibility acceleration up to hypervelocity regimes. Inspection of the dynamics of the system has shown some generic dynamical aspects that relate qualitatively to well known paradigms of stochastic acceleration. Scaling laws have been obtained that can be applied for quick assessment of experimental situations.

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